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STUDY OF PRETRANSITIONAL INDUCED BENARD CONVECTION BY TWO-DIMENSIONAL NUMERICAL EXPERIMENTS

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NOMENCLATURE

- Pr, Prandtl number ($Pr \equiv \nu/\kappa$);
- Ra, Rayleigh number;
- Ra^{cr}, critical value of Rayleigh number;
- $\tilde{V}$, convective velocity;
- V_0, velocity amplitude at $\varepsilon = 1$;
- V_P, triggering velocity;
- V_Z, vertical component velocity;
- X, coordinate in the horizontal plate;
- Z, coordinate in the vertical plate.

Greek symbols

- ε, reduced deviation of the Rayleigh number to the critical one [$\varepsilon \equiv (Ra - Ra^{cr})/Ra^{cr}$];
- θ, temperature perturbation amplitude;
- ν, kinematic viscosity;
- ξ, characteristic length;
- τ, characteristic time;
- Φ, vorticity;
- Ψ, stream function.

1. INTRODUCTION

IN A RECENT paper Wesfreid *et al.* [1] observed the induction of subcritical, space damped rolls by a triggering velocity applied to one side of a fluid layer maintained in controlled subcritical conditions, i.e. $\varepsilon = (Ra - Ra^{cr})/Ra^{cr} < 0$.

They compare their results with the Landau model expressed as:

$$\tau_0 \frac{\partial \tilde{V}}{\partial t} = \varepsilon \tilde{V} - \frac{\tilde{V}^3}{V_0^2} + \xi_0^2 \frac{\partial^2 \tilde{V}}{\partial X^2}, \tag{1}$$

where τ_0 and ξ_0 are the characteristic time and length respectively. V_0 is the velocity amplitude at $\varepsilon = 1$. At the stationary state for slow velocities [$\tilde{V}^3/(V_0^2|\varepsilon|) < 1$], the solution of the linearized equation (1) is

$$\tilde{V}(X) = V_P \exp - X/\xi^-, \tag{2}$$

with $\xi^- = \xi_0|\varepsilon|^{-0.5}$. V_P is the triggering velocity and

$$\lim_{x \to \infty} \tilde{V} = 0.$$

They verified this spatial exponential decay for the horizontal component of the velocity amplitude, V_X, of the induced rolls. They obtained a critical influence length $\xi^- = 0.378 d|\varepsilon|^{-0.505}$ in very good agreement with the theoretically expected law: $\xi^- = 0.385 d|\varepsilon|^{-0.5}$.

2. DESCRIPTION OF THE NUMERICAL EXPERIMENT

Recently Legros and Platten [2] gave new numerical results on nonlinear study of temperature and velocity distributions in the two-dimensional Bénard problem. This numerical approach has the particularity to simulate a Bénard apparatus of a given aspect ratio L/d with four rigid boundaries into which the number of convective cells is not imposed. The horizontal rigid boundaries are infinitely heat conductive and there is no heat flux through the lateral ones.

In this paper, we present results obtained with the finite differences method used in [2], for the study of pre-transitional induced Bénard convection. The dimensionless equations to be integrated are the conservation laws of momentum and of energy for a Boussinesq fluid of Prandtl number $Pr = 0.1$ in an apparatus of aspect ratio = 10.0. These equations are written as:

$$Pr \frac{\partial \Phi}{\partial t} = Pr \frac{\partial(\Psi, \Phi)}{\partial(X, Z)} - Ra \frac{\partial \theta}{\partial X} + Pr \nabla^2 \Phi, \tag{3}$$

$$Pr \frac{\partial \theta}{\partial t} = Pr \left[\frac{\partial(\Psi, \theta)}{\partial(X, Z)} - \frac{\partial \Psi}{\partial X} \right] + \nabla^2 \theta, \tag{4}$$

$$\Phi = \nabla^2 \Psi \tag{5}$$

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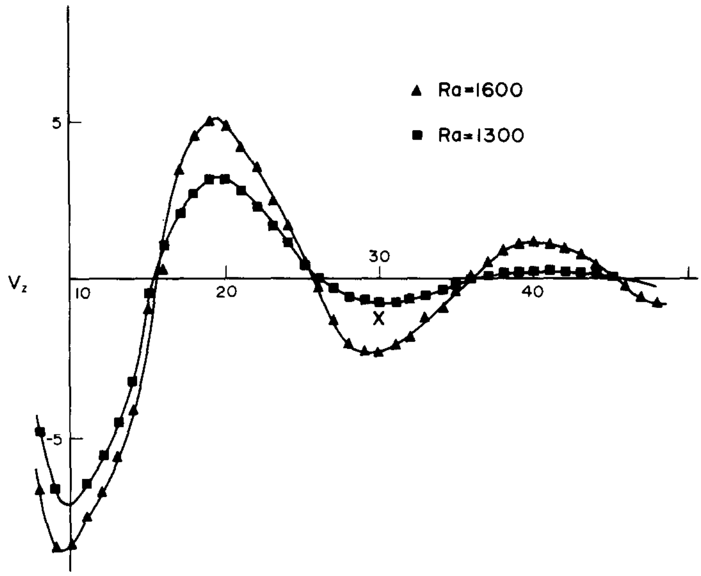

FIG. 1. The absolute value of the vertical component of the velocity $|V_Z|$ as a function of X in the induced rolls for $Ra = 1600$ and 1300. The first triggering rolls are not represented.

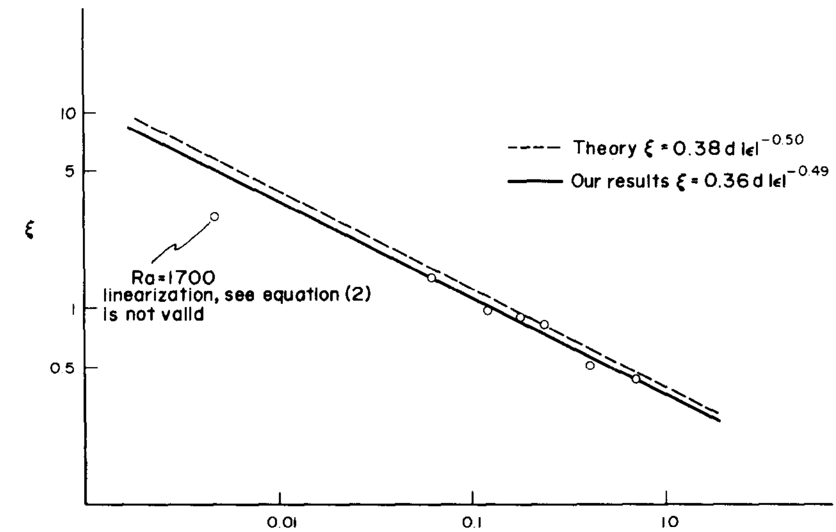

FIG. 2. The length of penetration as a function of ε for $Ra = 1700$, 1600, 1500, 1400, 1300, 1000, 500 estimated from $|V_Z|$.

and thus

$$\mathbf{v} = (v_X, v_Z) = \left(\frac{\partial \Psi}{\partial Z}, -\frac{\partial \Psi}{\partial X}\right). \tag{6}$$

In a first run, the Rayleigh number is fixed equal to 3500 and a small perturbation is imposed to the system. The numerical integration is continued until the steady state is obtained. For the other runs, the initial condition corresponds to a subcritical Rayleigh number and a state of rest everywhere, except for the grid points in the left part of the simulated apparatus which belong to the first roll. The stream function Ψ and the temperature values θ at each grid point in this first roll are put equal to those ones coming from the first run with $Ra = 3500$ and are not allowed to vary during the numerical integration. Details on the numerical procedure can be found elsewhere [3], [4]. Induced rolls spread out in the whole apparatus.

3. RESULTS

Values of the temperature θ and of the stream function Ψ at each grid point i,j of the discretized two-dimensional space (we used a grid 100×14) have been obtained for different values of ε corresponding to subcritical conditions.

In Fig. 1, we show at the steady state two systems of rolls whose vertical velocity amplitudes decay almost exponentially with X. The first roll at the left imposes a triggering velocity. This experiment was repeated for different values of Ra. One can see on Fig. 2, the effect of ε on the extension of the penetration convection. We obtained for the dependence $\xi^- = f(\varepsilon)$ calculated from the values of the vertical component of the velocities at $Z = 1/2d$:

$$\xi^- = 0.36d|\varepsilon|^{-0.49}. \tag{7}$$

This is in a good agreement with the results given in Section 1 for V_X. We should like to note here, that the X

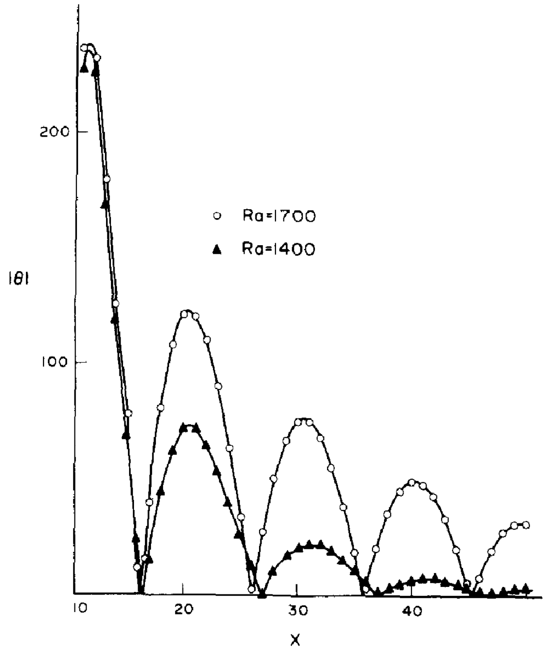

FIG. 3. Temperature perturbation θ in the induced rolls as a function of X is presented for $Ra = 1700$ and 1400. The first triggering rolls are not represented.

dependence of the successive maxima of Ψ has to obey in a first approximation the same law as V_Z [5].

In the same manner, an influence length can be deduced from the temperature perturbation amplitudes. The temperature perturbation amplitudes θ in the midplane of the simulated cell are plotted as a function of X in Fig. 3, for two different values of ε. We write in similarity with equation (2)

$$\tilde{\theta}(X) = \theta_M \exp - X/\xi^-, \tag{8}$$

where θ is the amplitude of the temperature perturbation distribution, θ_M is the maximum temperature perturbation, occurring at the limit between the triggering cell and the first induced one. In order to verify the spatial exponential decay in the temperature perturbation amplitude of the induced rolls, we plotted $\ln \theta$ vs X. This yields the value of ξ^- for a particular ε. The experiment is repeated for Ra = 1700, 1600, 1500, 1400, 1300, 1000, 500. For each ε we determine ξ; the values are plotted on Fig. 4 and obey the law

$$\xi = 0.37 d |\varepsilon|^{-0.48}. \tag{9}$$

This obtained dependence is in very good agreement with the results given in Section 1 from the velocity distribution. Our results obtained from the direct integration of the Boussinesq equations, confirm the validity of the Landau–Hopf model for this problem.

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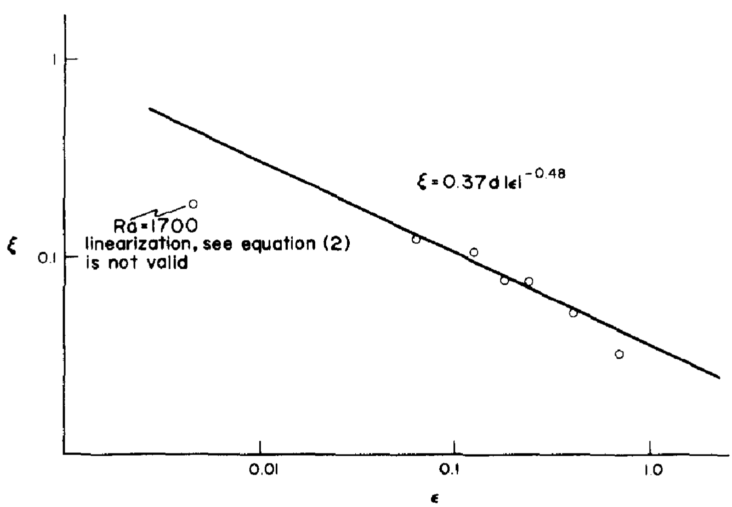

FIG. 4. The length of penetration as a function of ε for Ra = 1700, 1600, 1500, 1400, 1300, 1000, 500 estimated from θ.